The Chaotic Wheat Producer Price Growth Model

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Abstract: Movements in wheat producer prices are indicators of changes in the fundamentals of supply and demand. This paper creates the chaotic wheat producer price growth model and explains the local stability of the wheat producer price in the period 1991-2020 in the U.S., Germany and the Russian Federation. This paper confirms the stable growth of wheat producer price in those countries in the observed period.

1. INTRODUCTION

Guegan (2009) focuses on the use of dynamical chaotic systems in Economics and Finance. According to Guegan (2009), chaotic systems are complex systems that belong to the class of deterministic dynamical systems.

Namely, mathematicians were the first to be interested in this theory. In biology, scientists have used chaotic deterministic systems since the 1970s (May 1976). In physics, it is a long tradition for researchers to use these models. In economics, people working on stability and instability have flirted with bifurcation theory since the 1980s. Between the years 1986 and 1998, a lot of studies in economics used chaotic systems, following the idea of Grandmont (1988) (Guegan, 2009).

Chaos theory started with Lorenz’s model (1963). Chaos theory has been applied in economics by Day (1983), Goodwin (1990), Medio (1993), Lorenz (1993), Jablanovic (2022), etc.

Stability of motion and chaos theories can detect sensitivity to initial conditions. Tapia Cortez et al. (2018) explain mineral commodity prices dynamics. Their paper examines the chaotic behavior of annual copper prices between 1900 and 2015. They combine chaos theory, stability of motion and statistical techniques to reconstruct the long-term dynamics of copper prices. Their study recommends that the use of chaotic behavior improves our understanding of mineral commodity markets. In this sense, it improves the performance of traditional techniques for selecting key factors that influence market dynamics (Tapia Cortez et al., 2018).

The basic aim of this paper is to construct the chaotic wheat producer price growth model. The model is estimated.
2. THE MODEL

The chaotic wheat producer price growth model is presented by the following equations:

\[ D_t = \alpha - \beta P_t, \alpha > 0, \beta > 0 \]  \hspace{1cm} (1)

\[ S_t = -\gamma + \delta P_e, \gamma > 0, \delta > 0 \]  \hspace{1cm} (2)

\[ \frac{P_{t+1} - P_t}{P_t} = \mu (D_t - S_t), \mu > 0 \]  \hspace{1cm} (3)

\[ P_t = P^e \]  \hspace{1cm} (4)

Where \( P_t \) is the wheat producer price; \( P^e \) – the expected wheat producer price; \( S_t \) – supply function of wheat; \( D_t \) – demand function for wheat; \( \mu \) – the adjustment coefficient; \( \alpha, \beta \) – the coefficients of the demand function for wheat; \( \gamma, \delta \) – the coefficient of the supply function of wheat. (1) defines demand function for wheat; (2) defines supply function of wheat; (3) determines the relation between wheat producer price growth rate and surplus of demand for wheat; (4) explains relation between producer price for wheat and the expected producer price for wheat. By substitution one derives:

\[ P_{t+1} = [1 + \mu (\alpha + \gamma)] P_t - [\mu (\beta + \delta)] P_t^2, \quad \alpha, \beta, \mu, \gamma, \delta > 0 \]  \hspace{1cm} (5)

It is important to introduce \( p \) as \( p = P / P^m \), where \( p \) range between 0 and 1. \( P^m \) is the maximal size of the producer price in its time series. Further, growth rate of the producer price of wheat is obtained as:

\[ p_{t+1} = [1 + \mu (\alpha + \gamma)] p_t - [\mu (\beta + \delta)] p_t^2, \quad \alpha, \beta, \mu, \gamma, \delta > 0 \]  \hspace{1cm} (6)

This model given by equation (6) is called the logistic model. It is possible to show that iteration process for the logistic equation

\[ z_{t+1} = \pi z_t (1 - z_t), \pi \in [0,4], z_t \in [0,1] \]  \hspace{1cm} (7)

is equivalent to the iteration of growth model (6) when we use the identification

\[ z_t = \frac{\mu (\beta + \delta)}{[1 + \mu (\alpha + \gamma)]} p_t \quad \text{and} \quad \pi = [1 + \mu (\alpha + \gamma)] \]  \hspace{1cm} (8)

Using (6) and (8) we obtain:

\[ z_{t+1} = \mu (\beta + \delta) p_t \left( 1 - z_t \right) \]  \hspace{1cm} (9)

On the other hand, using (7) and (8) we obtain:

\[ z_{t+1} = \mu (\beta + \delta) p_t \left( 1 - z_t \right) \]  \hspace{1cm} (10)
It is obtained that: (i) For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$; (ii) For $1 < \pi < 3.57$ there exist fixed points the number of which depends on $\pi$; and (iii) For $3.57 < \pi < 4$ the solution become “chaotic”.

3. EMPIRICAL EVIDENCE

The main aim of this analysis is to present the producer price growth stability in USA, Russian Federation, Germany in the observed periods by using the logistic model (9). In this sense,$$
p_{t+1} = \pi p_t - \nu p_t^2, \quad \pi \in [0, 4]
$$
where $p$ – producer price of wheat, $\pi = [1 + \mu (\alpha + \gamma)]$ and $\nu = \mu (\beta + \delta)$. Now, we estimate the model (9). The results are presented in Tables 1-3 (FAO, 2022).

Table 1. The estimated model (9): Wheat Producer price (USD/tons), 1991-2020, USA

<table>
<thead>
<tr>
<th>R=0.82504 Variance explained 68.069%</th>
<th>$\pi$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>1.239126</td>
<td>0.348518</td>
</tr>
<tr>
<td>Std. Err.</td>
<td>0.124734</td>
<td>0.171399</td>
</tr>
<tr>
<td>$t(27)$</td>
<td>9.934113</td>
<td>2.033373</td>
</tr>
<tr>
<td>p-level</td>
<td>0.000000</td>
<td>0.051952</td>
</tr>
</tbody>
</table>


Because $\pi = 1.239126$, then producer price of wheat monotonically increased in the period 1991-2020, in the USA.

Table 2. The estimated model (9): Wheat Producer Price (USD/tons) Annual value, 1992-2020, Russian Federation

<table>
<thead>
<tr>
<th>R=0.62878 variance explained 39.536%</th>
<th>$\pi$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>1.437326</td>
<td>0.714277</td>
</tr>
<tr>
<td>Std. Err.</td>
<td>0.18665</td>
<td>0.251491</td>
</tr>
<tr>
<td>$t(26)$</td>
<td>7.00671</td>
<td>2.840163</td>
</tr>
<tr>
<td>p-level</td>
<td>0.000000</td>
<td>0.008644</td>
</tr>
</tbody>
</table>


Because $\pi=1.437326$, then producer price of wheat monotonically increased in the period 1992-2020, in Russian Federation.

Table 3. The estimated model (9): Wheat Producer Price (USD/tons) Annual value, 1991-2020, Germany

<table>
<thead>
<tr>
<th>R=0.72285 Variance explained 52.851%</th>
<th>$\pi$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>1.305768</td>
<td>0.451004</td>
</tr>
<tr>
<td>Std. Err.</td>
<td>0.14943</td>
<td>0.199413</td>
</tr>
<tr>
<td>$t(27)$</td>
<td>0.734643</td>
<td>2.261655</td>
</tr>
<tr>
<td>p-level</td>
<td>0.000000</td>
<td>0.31981</td>
</tr>
</tbody>
</table>


Because $\pi=1.305768$, then producer price of wheat monotonically increased in the period 1991-2020, in Germany.
Figure 1. Wheat Producer Price (USD/tons), 1991-2020, USA

Figure 2. Wheat Producer Price (USD/tons) Annual value, 1992-2020, Russian Federation

Figure 3. Wheat Producer Price (USD/tons) Annual value, 1991-2020, Germany
4. CONCLUSION

This paper creates the chaotic producer wheat price growth model. Also, this paper confirms that the coefficient \( \pi = [1+\mu (\alpha+\gamma)] \) plays a crucial role in explaining the local stability of the producer wheat price, where, \( \mu \) is the adjustment coefficient; \( \alpha \) is the coefficient of the demand function for wheat, \( \gamma \) is the coefficient of the supply function of wheat. The estimated values of the coefficient \( \pi \) were between 1 and 2 in the USA, Russian Federation and Germany in the observed periods. These results confirm a stable growth of producer wheat price in those countries in the observed periods.

References

Tapia Cortez, C.A., Coulton, J., Sammut, C., & Saydam, S. (2018). Determining the chaotic behaviour of copper prices in the long-term using annual price data. *Palgrave Communications*, 4, 8. [https://doi.org/10.1057/s41599-017-0060-x](https://doi.org/10.1057/s41599-017-0060-x)